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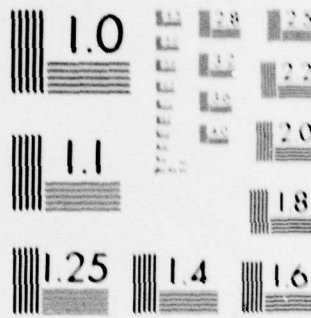   MAY 79   J T LO   AFOSR-74-2671
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Final Scientific Report . 1 Jun 74 —

on

Detection, Estimation, and Control

on Group Manifolds

TO: Air Force Office of Scientific Research

Air Force Systems Command

USAF

31 May 79

FROM: James T. Lo

Department of Mathematics

University of Maryland Baltimore County

Baltimore, Maryland 21228

SUBJECT: Grant No. AFOSR-74-2671

Detection, Estimation, and Control

on Group Manifolds

covering the period:

June 1, 1974 - May 31, 1979

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TABLE OF CONTENTS

I. General

This final report covers work carried out by faculty and staff members of the Department of Mathematics during the five-year period 1 June 1974 through 31 May 1979 under Grant No. AFOSR-74-2671.

Progress was mainly contained in the technical papers and reports listed in Section II. An interim report was submitted to AFOSR at the end of each of the first four years.

During the report period, the following people contributed to the project: Associate Professor James T. Lo, Dr. Linda R. Eshleman, Dr. Masahiro Nishihama, and Dr. Shirish Chikte.

II. Publications

(1) Global Bilinearization of Systems with Control Appearing Linearly, SIAM Journal on Control, 13-4, pp. 879-885, July 1975.

(2) Eigenvector Placement in Linear Multivariable Systems, UMBC Mathematics Research Report No. 74-13, UMBC, Baltimore, Md., September 1974.

(3) Signal Detection for Bilinear Systems, UMBC Mathematics Research Report No. 73-7, UMBC, Baltimore, Md., November 1973, Information Sciences, 9, pp. 249-278, 1975.

(4) Exponential Fourier Densities and Estimation and Detection on a Circle, UMBC Mathematics Research Report No. 76-1, UMBC, Baltimore, Md., January 1976, also IEEE Transactions on Information Theory, IT-23, pp. 110-116, January 1977.

(5) Estimation for Discrete-Time Directional Processes, UMBC Mathematics Research Report No. 75-11, UMBC, Baltimore, Md., October 1975, also Proc. of the Sixth Symposium of Nonlinear Estimation and its Applications, pp. 171-180, September 1975 (with L. R. Eshleman).

(6) Exponential Fourier Densities on S^2 and Optimal Estimation and Detection for Directional Processes, UMBC Mathematics Research Report No. 76-8, UMBC, Baltimore, Md., July 1976, also IEEE Transactions on Information Theory, vol. IT-23, No. 3, pp. 321-336, May 1977 (with L. R. Eshleman).

(7) Exponential Fourier Densities on SO(3) and Optimal Estimation and Detection for Rotational Processes, UMBC Mathematics Research Report No. 76-9, UMBC, Baltimore, Md., Invited paper, Proc. of the 1976 IEEE Conference on Decision and Control, Clearwater Beach, Florida, pp. 134-140, December 1976, also scheduled for publication in SIAM Journal on Applied Mathematics, Part A, February 1979 (with L. R. Eshleman).

(8)   Exponential Fourier Densities and Optimal Estimation for Axial
       Processes, UMBC Mathematics Research Report No. 77-1, UMBC,
       Baltimore, Md., January 1977, also IEEE Transactions on Information
       Theory, IT-25, No. 4, July 1979.

 (9)   Estimation and Detection on Lie Groups, Invited survey paper to
       appear in Nonlinear Estimation and Filtering Theory: A Status
       Review, edited by E. B. Stear, Marcel Dekker, 1979.

(10)   Estimation Problems with Lie Group Structure, Invited paper, Proc.
       of the 1977 IEEE Conference on Decision and Control, pp. 634-638,
       December 1977.

(11)   Optimal Filters for Nilpotent Associate-Algebraic Bilinear Systems,
       Proc. 6th Annual Allerton Conference on Communication, Control,
       and Computing, Monticello, Illinois, October 1978 (with Shirish
       Chikte).

(12)   Optimal Filters for Bilinear Systems with Nilpotent Lie Algebras,
       UMBC Mathematics Research Report No. 78-13, UMBC, Baltimore, Md.,
       to appear in IEEE Transactions on Automatic Control (with Shirish
       Chikte).

(13)   Optimal Estimation for the Satellite Attitude using Start Tracker
       Measurements, UMBC Mathematics Research Report No. 78-14, UMBC,
       Baltimore, Md., also presented at the 1978 NASA Symposium on
       Flight Mechanics/Estimation Theory, Goddard Space Flight Center,
       Greenbelt, Md., October 1978.

(14)   Convolution of Exponential Fourier Densities and Recursive Filtering
       on the Circle, presented at the 1979 IEEE Conference on Decision
       and Control, San Diego, California, January 1979 (with M. Nishihama).

III. Summary of Progress

The results covered by this final report can be put in two categories -
continuous-time systems and discrete-time systems. The continuous-time
systems considered are bilinear in form. The bilinear systems have been
extensively studied in recent years for three primary reasons. First, it has
been shown that bilinear systems are feasible mathematical models for large
classes of problems of practical importance. Second, bilinear systems
provide higher order approximations to nonlinear systems than do linear systems.
Third, bilinear systems have rich geometric and algebraic structures which
promise a fruitful field of research.

One of our main results gave a necessary and sufficient condition that a
nonlinear system, with control appearing linearly, be dynamically equivalent
to an observable bilinear system. When the condition is satisfied, a

procedure to construct an observability canonical form of such a bilinear system was provided.

Many interesting detection problems including the demodulation for a frequency-shift-keyed binary communication system involve bilinear systems. Such problems were studied by us from the proposed group-manifold viewpoint. A likelihood ratio formula was derived as a function of up-dated observation.

A least-squares filtering problem in which the signal process to be estimated is generated by a nilpotent bilinear system was studied. It was found that the filtering equations are also bilinear in form and possesses the nilpotency property. By means of an illustrative example, it was shown how the dimensionality of the filter may be reduced by eliminating certain inherent redundancies.

Perhaps, the most important contribution made by us during the report period is the discovery and development of the exponential Fourier densities on Lie groups for the discrete-time systems. The EFD's were found to have three desirable properties that provide us with a firm grip on discrete-time detection and estimation problems on group manifolds. First, any continuous or bounded-variation density on a compact Lie group can be approximated as closely as desired by an EFD. Second, the EFD's of a given order are closed under the operation of taking conditional distributions. Third, they are "almost" closed under convolution.

All these properties and their applications to estimation and detection on S^1, $SO(3)$, S^2, $S^2/\pm1$, and general compact Lie groups were reported in most of the publications listed in Section II, which reflect that about 65% of our research effort supported by the Grant was made to develop the full strength of such densities. Above all, a practical application of the EFD's to the satellite attitude determination using star tracker measurements was carried out during the report period. The application represents a new approach to spacecract attitude determination and, perhaps, the first optimal nonlinear filter ever implemented for a real-world system.

We will, in the following, summarize the ideas and results of each of the publications.

(1) Global Bilinearization of Systems with Control Appearing Linearly --

Bilinear systems have been extensively studied in recent years for three primary reasons. First, it has been shown that bilinear systems are feasible

3

mathematical models for large classes of problems of practical importance. Second, bilinear systems provide higher order approximations to nonlinear systems than do linear systems. (Linear systems are special cases of bilinear systems.) Third, bilinear systems have rich geometric and algebraic structures which promise a fruitful field of research.

Perhaps this paper can provide the fourth reason for studying bilinear systems. It is shown in the paper that a large class of control systems with highly nonlinear features can be _transformed_ into bilinear systems. A necessary and sufficient condition that a nonlinear system, with control appearing linearly, be dynamically equivalent to an observable bilinear system is derived. When the condition is satisfied, a procedure to construct an observability canonical form of such a bilinear system is provided in the proof of the sufficiency part of the condition.

A precise statement of the main result is given as a theorem on page 881 of the paper.

(2) Eigenvector Placement in Linear Multivariable Systems --

A very important system design problem may best be explained by the following _hypothetical_ example of jet engine control: The control mechanism of a jet plane, which is composed of an electronic analog computer (or a digital computer) and a mechanical actuator, should be designed to meet two main goals. First, the mechanism should be able to shift every relevant component of the engine and the airplane body into its best operation level for each possible maneuver. Second, the action required of the pilot to accomplish each maneuver should be made as simple as possible. As many feats can be done by linear autonomous systems, it is not too restrictive to assume that such a system can be used to accomplish the first goal. However, feeding input appropriately may be too large a job for the pilot, particularly if there are a large number of input terminals. Therefore, how to reduce the external control terminals that are directly maneuvered by the pilot without affecting the airplane performance is a crucial design problem.

This is exactly the kind of problem studied in this paper. A necessary and sufficient condition is given for reducing the number of external input terminals without affecting the controllability property of the linear system. When this condition is satisfied, an output feedback can be computed by an algorithm given in the paper. This output feedback gain shows how to

4

hook up the output and the undesired input terminals, and thus reduce the operating burden of the pilot.

Mathematically speaking, this paper presents a necessary and sufficient condition for the existence of a linear output feedback law which shifts the system eigenvectors out of an arbitrary linear subspace. When the condition is satisfied, a procedure to construct such an output feedback law is given in the proof of the condition.

(3) Signal Detection for Bilinear Systems --

Many interesting detection problems including the demodulation for a frequency-shift-keyed binary communication system involve bilinear systems. This paper studies such problems from the proposed group-manifold viewpoint.

The injection operator used by McKean to construct Brownian paths on a Lie group is employed to formulate a class of signal detection problems on matrix Lie groups. The hypotheses that the signal is absent and present in the observation on a Lie group are described by a pair of bilinear matrix Ito equations. The injection operator is shown to be almost surely bijective and its inverse is constructed. This bijection enables us to do detection and estimation of processes taking values on a Lie group in a finite dimensional linear space--the Lie algebra. In fact, bilinear detection on Lie groups ("locally linear spaces") is in essence equivalent to linear detection on linear spaces. A likelihood ratio formula for bilinear matrix Ito equations is thus derived as a function of up-dated observation.

(4) Exponential Fourier Densities and Estimation and Detection on a Circle --

In a series of recent papers, the Fourier series is used to represent probability densities in studying estimation on the circle, $S^1 = [-\Pi, \Pi]$. Many suboptimal estimation schemes have been obtained by truncating the higher frequency terms as necessitated in any numerical equation. However, simulation results indicate that the approach is not fully satisfactory.

One intuitive explanation is that the higher order harmonic terms need not be negligible, as evidenced by the formal Fourier expansion, $p(x) = \frac{1}{2\Pi} + \frac{1}{\Pi} \sum_{n=1}^{\infty} \cos n(x - x_0)$, of the unit mass density at $x = x_0$. As a matter of fact, applying the Bayes rule to calculate a conditional density always involves the multiplication of two a priori densities, and the multiplication of two Fourier series has the effect of spreading the dominent Fourier coefficients into higher frequency terms. Consequently, the approach

5

places severe limitations on the performance of the suboptimal schemes obtained from truncating the higher frequency terms. Obviously, the dilemma is especially serious in multistage estimation when a sequence of multiplications of Fourier series takes place.

The purpose of the paper is to propose an alternative approach to avoid this difficulty. The underlying idea of the approach is very simple. A probability density function of S^1 can be viewed as a periodic function on the real line. As the function is nonnegative, it can be shown that the function can be approximated as closely (to be specified later) as we wish by a function in the form,

$$\exp \sum_{k=0}^{n} (A_k \cos kx + b_k \sin kx)$$

which we will call an exponential Fourier density of the n-th order. It is obvious that the multiplication of two such densities does not raise the order of the densities. Thus the class of all n-th order Fourier densities is closed under the operation of taking conditional distributions. We recall that a property of Gaussian densities responsible for the simple K-B filtering is exactly such a closure property. Therefore, it is not a surprise that the use of exponential Fourier densities leads to simple resolutions of many estimation problems on the circle.

In this paper a basic theorem for approximating a probability density with an exponential Fourier density is given. Three models are then formulated and studied which are believed to be generic in practical problems of detection and estimation. Optimal recursive schemes are derived for these models, thereby illustrating how the closure property mentioned above facilitates finite-dimensional, closed-form, and exact solution.

(5) Estimation for Discrete-Time Directional Processes --

In many problems of practical importance, we are concerned with three dimensional directions. A notable engineering example of this kind is the earth-pointing satellite attitude determination and control. Some other examples can be found in orientation analysis in biology, dip and declination study in geology, seasonal fluctuation phenomena in medicine and wind direction forecast in meteorology.

It is not pretended that in this paper we resolve any specific estimation problems involved in these practical subjects. The purpose of this paper

6

is rather to propose a simple mathematical model which is believed to be generic in many such situations involving directional data, and, by detailed analysis of the model, to illustrate the structures of multistage estimation for directional processes. We call a process, whose state space is a circle a sphere, or a hypersphere, a directional process.

In this paper we are mainly concerned with three-dimensional directional processes, processes on a sphere. Estimation for higher dimensional directional processes can be treated similarly. Estimation on a circle is substantially simpler. In fact the approach used here is an (nontrivial) extension of that used in (4).

Error criteria, probability distributions, and optimal estimates on a sphere are discussed. Exponential Fourier densities on a circle introduced in (4) are generalized to those on a sphere. The underlying idea for using the exponential Fourier densities is the same as that on the circle in (4). Namely, the closure property of these densities under the operation of taking conditional densities enables us to compute the optimal estimates efficiently by updating only a fixed and finite number of parameters. In contrast to the circle case, this closure property does not exist for all combinations of signal and noise densities on the sphere. As a matter of fact, a key issue addressed in this paper is to characterize such combinations of signal and noise densities that have this closure property.

(6) Exponential Fourier Densities on S^2 and Optimal Estimation and Detection of Directional Processes --

Optimal estimation on S^2, the unit sphere, was first studied in [5] using exponential trigonometric densities (ETD's). The results in [5] have two distinct deficiencies: (1) The class of admissable noise densities is restricted. (2) The signal must be a constant direction. It is these deficiencies that motivated the continuing effort which results in the present paper.

The first deficiency led to the use of the class of exponential Fourier densities (EFD's) on S^2. Any continuous density can be approximated as closely as desired by an EFD. Furthermore, the EFD's are all admissable noise densities.

The second deficiency led to the use of three new kinds of displacement on S^2. Each kind can be used to construct one time-varying signal process and one observation process. Various combinations of the signal processes and the obser-

7

vation processes yield 16 different estimation models. The additive observation noise case is also considered. The merits and disadvantages of each model are discussed in detail, when recursive formulas are derived for the multistage conditional densities for many of the models.

In general, neither the class of ETD's nor the class of EFD's is closed under the operation of taking conditional distributions with respect to each of the models. An alternative usage of the classes is required to derive each of the recursive schemes for optimal estimation.

Signal detection problems for the models are also studied. Recursive algorithms for the likelihood ratios are obtained for many of the models.

In short, this paper gives a comprehensive and comparative treatment of many estimation and detection models on the sphere. We believe that it provides a chest of tools which will be useful in analyzing and synthesizing practical estimation and detection problems.

(7) Exponential Fourier Densities on SO(3) and Optimal Estimation and Detection of Rotational Processes --

Rigid body rotations are involved in many important practical problems of detection, estimation, and control. Some notable examples can be found in gyroscopic analysis and satellite attitude determination and control. While linearization and approximation techniques have led to many useful results, simple analytic tools which will enable us to analyze and synthesize the optimal structures have long been desired.

Optimal estimation and detection schemes for discrete-time processes whose state space is a circle or sphere have been obtained in (4) and (6) by using a novel representation for probability densities which has the form exp f where f is a finite linear combination of functions which form a complete orthogonal system on the state space involved. In the case of the circle, circular functions were used, while both spherical harmonics and multiple trigonometric functions were employed for densities defined on the sphere.

In this paper the same approach will be taken for discrete-time rotational processes by introducing a similar exponential density referred to as a rotational exponential Fourier density (REFD) defined on the group of rotations of three-dimensional space, that is obtained by using a sequence of irreducible unitary representations which form a complete orthogonal system on SO(3). It can be shown that a continuous density function on SO(3) can be

8

approximated by such a REFD as closely as we wish in the space of square integrable functions.

(8) Exponential Fourier Densities and Optimal Estimation for Axial Processes --

In this paper we consider the problem of estimating axes in three-dimensional space. An axis or axial vector is distinguished from a polar vector in that the former is invariant under inversion. Such axes occur in many diverse areas including the following: geophysical fluid dynamics to estimate the vorticity of a flow, paleomagnetism to estimate a magnetic field, crystallography to estimate the optic axis of a crystal, geology to estimate the direction of a normal to the axis of a fold in a layer of rock, and quantum mechanics to estimate the axis of rotation of a rigid body rotation.

Using densities of the form exp f where f is a linear combination of axially symmetric spherical harmonics, estimation problems which arise by examining various possible ways of obtaining a displacement of an axis will be solved in this paper. Although the state space under consideration is homeomorphic to a hemisphere of S^2, the results for estimation on S^2 cannot be applied for several important reasons: the displacements defined in that paper may result in a given point being displaced to a non-antipodal point in the opposite hemisphere, the densities on S^2 were not, in general, axially symmetric, and the error criterion used for S^2 is undesirable since it would result in a rejection of the antipode of the optimal estimate.

Using the various displacements and conditional densities obtained in this paper, detection for axial processes would be described by procedures similar to those used for S^2 and SO(3).

(9) Estimation and Detection on Lie Groups --

There are five main sections in this survey chapter to be included in the forthcoming book, Nonlinear Filtering and Estimation Theory--A Status Review, edited by E. B. Stear. They are summarized as follows:

(9a). PROBABILITY ON THE CIRCLE.

There are many fundamental differences between the estimation and detection problems on the Euclidean spaces and those on the Lie groups. In order for some readers to appreciate them, this section will be addressed to some probabilistic elements on the circle. The probability distribution function and the

9

characteristic function on the circle will first be briefly introduced.

One of the main concerns in this chapter is to study how one uses the knowledge of the probability distribution of a random variable taking values on a Lie group to determine an estimate of the random variable that minimizes a certain error criterion. The conventional least squares technique cannot be used here. Let us take the circle as an example. The square error of the angles 0° and 359° is $(359^2)^{\circ}$, whereas by geometrical intuition they are only 1° apart. In the sequel we will look into this issue on the circle in detail.

The importance of the normal probability densities cannot be over-emphasized for estimation and detection on Euclidian spaces. Unfortunately, there does not exist an analogous density on the circle that possesses all the nice properties of the normal density. In fact, the nice properties of the normal density are almost equally divided between two contenders for normalcy, the folded normal density and the circular normal density. It turns out that while the folded normal density is natural to use for continuous-time estimation, the circular normal density is more suitable for discrete-time estimation. They will both be discussed and compared in this section.

(9b). DISCRETE-TIME ESTIMATION ON THE CIRCLE.

Estimation for discrete-time systems on the circle was studied before, using both folded normal densities and Fourier series representations of probability densities. The optimal estimation equations obtained therein are infinite-dimensional and cumbersome. Although some numerical simulation has been done on the suboptimal equations obtained from truncating the higher order terms, it is not clear whether these equations have satisfactory performance in general.

As a matter of fact, the "dimension" of the optimal estimation equations derived from using the folded normal densities increases very rapidly in time. When the Fourier series are used to represent probability densities, the application of Bayes' rule, which involves the multiplication of two a priori densities, has the effect of spreading the dominant Fourier coefficients into the higher order terms. Obviously, this dilemma becomes compounded in multistage estimation problem when a sequence of multiplications of Fourier series takes place.

In this section, we will present an alternative approach. The approach is based on a new class of probability density functions which have the form

$$\exp \sum_{k=0}^{n} (a_k \cos kx + b_k \sin kx).$$

Such a density will be called an exponential density of order n, to be denoted by EFD(n).

(9c). CONTINUOUS-TIME ESTIMATION ON THE CIRCLE.

A signal process and an observation process, taking values on S^1, will be formulated in terms of bilinear Ito matrix differential equations. The conditional probability distribution of the signal, given observations over a certain period of time, will be evaluated. Recursive computational schemes for optimal estimation (filtering, smoothing, and prediction), with respect to the error criteria defined in Subsection II.2, will be derived. In fact it will be shown that optimal estimates on S^1 can be obtained recursively by the use of an ordinary vector space estimator together with a nonlinear preprocessor and a nonlinear postprocessor. Multichannel estimation on abelian Lie groups will be examined. Examples illustrating the optimal estimation procedure are given at the end of this section.

(9d). DISCRETE-TIME ESTIMATION ON COMPACT LIE GROUPS.

The results of (2b) can be easily generalized to the problems on compact Lie groups by introducing a similar exponential Fourier density (EFD) on the group. This density is obtained by using a sequence of irreducible unitary representations which form a complete orthogonal system on the compact group. It can be shown that a continuous density function on the group can be approximated by such an EFD as closely as we wish in the space of square integrable functions.

As in the circle case, the class of ERD's of a certain finite order on the compact Lie group is closed under the operation of taking conditional distributions as a consequence of the group structure of the group. It will become clear in the sequel that it is exactly this closure property of the EFD's that yields simple estimation schemes which update the sequential conditional densities by recursively revising a finite and fixed number of parameters.

In order to illustrate how the conditional density can be used to calculate the optimal estimate on the group, a rigid body attitude estimation

11

problem is solved as an example. The error criterion, the optimal estimate, and the estimation error with respect to the criterion will be derived for a given probability distribution.

(9e). DETECTION FOR CONTINUOUS-TIME SYSTEMS ON LIE GROUPS.

The idea of "rolling without slipping" introduced in Section IV will now be generalized and used to formulate an observation process on an arbitrary matrix Lie group. Briefly speaking, we will inject the differentials of an observation process described by a vector Ito differential equation into a Lie group via the exponential map and then piece them together. The resulting product integral describes our observation process on the Lie group, the injected vector observation process being called its skew form.

The observation process thus constructed on a Lie group will be seen to satisfy a bilinear matrix stochastic differential equation, when its skew form is linear. The observational noise can be viewed as entering multiplicatively.

Given an arbitrary bilinear matrix observation process, we will show that the corresponding skew observation process can be obtained by "reversing" the above injecting procedure. Furthermore, these two procedures will be seen to induce two "almost sure" bijective mappings between a vector-valued and a matrix-valued function spaces, one being the inverse of the other.

It is well known that the study of a Lie group may be greatly simplified by considering the tangent space (the Lie algebra) of the Lie group at its identity. In fact, the local study of a Lie group is entirely equivalent to the study of the finite dimensional linear algebraic structures of the Lie algebra. In this paper, the above bijective mappings facilitate similar simplification. It enables us to evaluate the likelihood ratio in a finite dimensional linear space--the Lie algebra!

In view of the above construction, the null and the alternative hypotheses that the signal is respectively absent and present in the observation on a Lie group can be written as a pair of bilinear matrix stochastic differential equations. Using the bijective mappings, we may transform these hypotheses on a Lie group into those on the corresponding Lie algebra. There the likelihood ratio can be expressed by the well-known Duncan's formula. Thus the evaluation of the likelihood ratio on a Lie group also hinges on the least-squares estimation.

When the signal is a linear diffusion process, the idea of using the bijective mappings to work in the Lie algebra also leads to a finite dimensional filtering equation for evaluating the least-squares estimate. This equation is indeed an immediate extension of the Kalman-Bucy filter to the case with observation on Lie groups.

(10) Estimation Problems with Lie Group Structure --

The exponential Fourier densities were used to study estimation on the unit circle, the unit sphere, the three-dimensional rotation group, and the projective two-space in a sequence of recent papers. Many finite-dimensional optimal estimation schemes were obtained mainly due to the closure property of the exponential Fourier densities of any given finite order under the operation of taking conditional distributions. Another reason for using exponential Fourier densities is that any continuous or bounded-variation probability density on the aforementioned spaces can be very closely approximated by such a density.

It is the purpose of this paper to generalize the previous results to an arbitrary compact Lie group and thereby to illustrate that it is the structure of a compact Lie group that accounts for the usefulness of the exponential Fourier densities.

As it is expected that most readers of this paper will be engineers, some definitions and preliminary results will be briefly summarized to facilitate our presentation in the paper.

(11) Optimal Filters for Nilpotent Associate Algebraic Bilinear Systems --

In this paper least squares filtering problem is considered wherein the signal process of interest is generated by a bilinear dynamical system driven by a Gauss-Markov process while the observation process is generated by adding white, gaussian noise to the above Gauss-Markov process.

Explicit solution to the above nonlinear estimation problem is obtained when the matrix algebra associated with this bilinear equation is nilpotent; i.e., the produce of more than a certain fixed finite number of matrices in this algebra vanishes. It is shown that the resulting filter consists of a Kalman-Bucy filter followed by a bilinear (time-varying) system which also possesses the nilpotency property.

Due to the above features the filter is seen to be quite suitable for practical realization. On the other hand the dimensionality of the filter is quite high. By means of an illustrative example it is shown how the dimen-

13

sionality may be reduced by eliminating certain inherent redundancies.

The motivation for this paper comes from the belief that the class of bilinear signal processes considered here can approximate a much wider class of general nonlinear signal processes.

(12) Optimal Filters for Bilinear Systems with Nilpotent Lie Algebras --
This paper discusses a least squares filtering problem in which the signal process to be estimated is generated by a bilinear dynamical system. The problem is illustrated schematically in figure 1 below.
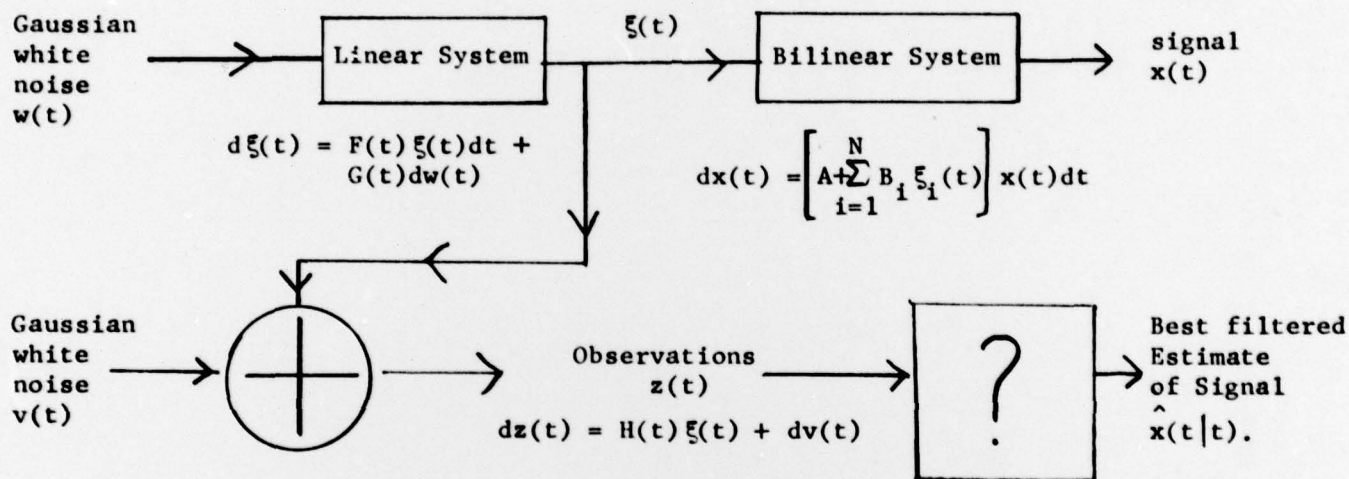
Gaussian white noise $w(t)$ → Linear System → $\xi(t)$ → Bilinear System → signal $x(t)$

$$d\xi(t) = F(t)\xi(t)dt + G(t)dw(t)$$

$$dx(t) = \left[A + \sum_{i=1}^{N} B_i \xi_i(t)\right] x(t)dt$$

Gaussian white noise $v(t)$ → ⊕ → Observations $z(t)$ → ? → Best filtered Estimate of Signal $\hat{x}(t|t)$.

$$dz(t) = H(t)\xi(t) + dv(t)$$

figure 1

A Block Schematic of the Nonlinear Filtering Problem

An exact constructive solution to the above problem is provided under the assumption that the <u>Lie algebra</u> associated with the bilinear system is <u>nilpotent</u>. The resulting filter turns out to be of the form shown in figure 2.
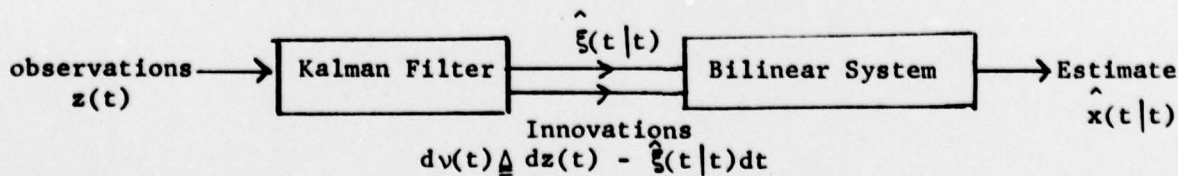
observations $z(t)$ → Kalman Filter → $\hat{\xi}(t|t)$ → Bilinear System → Estimate $\hat{x}(t|t)$

Innovations
$$d\nu(t) \triangleq dz(t) - \hat{\xi}(t|t)dt$$

figure 2

Optimal Filter Structure

14

Moreover the bilinear system in the filter structure also possesses the nilpotency property.

The dimensionality of the above filter is formidable and some ways of reducing it are demonstrated via an illustrative example. Finally three important special cases are studied in which the bilinear system in figure 2 can be replaced by a simple, memoryless nonlinearity. Possible application of this work include estimation of rotational motion of rigid bodies with one degree of freedom.

(13) Optimal Estimation for the Satellite Attitude Using Star Tracker Measurements --

This paper is mainly concerned with estimating the satellite attitude given the gyro readings and the star tracker measurements of a commonly used satellite attitude measuring unit (SAMU). The SAMU is used in such satellites as the high energy astronomy observatory (HEAO) and the precision pointing control system (PPCS). It is composed of 3 to 6 rate gyros and 2 star trackers. The satellite attitude is propagated over a certain number of small time intervals by integrating the satellite angular rates determined from the gyro reading. Gyro drift rates, misalignments, and lack of a precise initial attitude reference then make it necessary to employ two gimbaled star trackers to provide a bench mark to the further propagation of the satellite attitude. A star tracker utilizes an image dissector tube to locate the position of a star on its photosensitive surface. Due to the non-stationary nonlinear characteristics of the image dissector deflection coils and the white noise from the processing electronics, it is at this stage that estimation is required.

A new representation of a probability density of a three dimensional rotation called the exponential Fourier density (EFD), was recently introduced which has the desirable closure property under the operation of taking conditional distributions. Using the EFD's, an approach was suggested to derive recursive formulas for updating the conditional densities of a rotational process given a nonlinear observation in additive white noise.

In this paper, this approach is carried out for the aforementioned satellite attitude estimation problem. The recursive formulas for updating the conditional densities of the satellite attitude are derived for arbitrary star tracker equations. These general formulas are included here to accom-

15

modate possible future consideration of the distortion characteristics of the image dissector deflection coils and possible future change in the star tracker configuration. These general formulas also provide a basis on which special cases can be easily analyzed. However, they involve a large amount of computation. Their feasibility for on-board implementation is highly questionable.

In a conversation with E. J. Lefferts of the GSFC/NASA, it was observed by him that by choosing appropriately the mathematical description of the star tracker configuration, the star tracker measurement can be expressed in closed form as a linear combination of the rotational harmonic functions of order one. This observation substantially simplifies the optimal estimation scheme and greatly reduces the amount of computation required in both design-ing and utilizing the scheme. A detailed derivation of the associated equations is included in the paper.

A close look at the mathematical models for the star tracker measurement revealed that the measurement models are not observable. The nonobservability causes a pseudo-image of each observed star. As the extended K-B filtering is merely a local processing, it does not pick up the pseudo-image and is therefore immune from its effect. In contrast, the optimal scheme does not have any "blind spots" and thus assigns an equal probability to the double images of each of the observed stars. An example illustrating such nonobservability is given.

Fortunately enough, this difficulty resulting from the nonobservability can be remedied by introducing a "pseudo-measurement" of the second apparent star direction cosine $u_2(k)$ with respect to the tracker base reference axes. We note that $u_2(k)$ is the component of the direction vector $u(k)$ that is perpendicular to the tracker field-of-view and hence cannot be measured directly by the tracker. However, from using the satellite attitude estimate $\hat{s}(t)$ at the previous step $t=k-1$, $u_2(k)$ can be predicted, which is to be used as a "measurement" of the real $u_2(k)$. Facilitated with such pseudo-measurement, the pseudo-image of the observed star can be eliminated. For want of a mathematically rigorous proof, only a heuristic explanation for this pseudo-measurement approach, which is believed to be new, is given in the paper.

The computation required to produce the optimal estimate, involves integrating the conditional covariance matrix of the attitude quaternion, which is very CPU-time-consuming. Encouraging is the fast and high con-

centration of the conditional probability density at the satellite attitude under estimation. This phenomenon is dictated by the theory and suggests two possible ways to get around the difficulty of integration. One way is to localize the integration. Another way is to use the maximum likelihood estimate instead of the optimal estimate. Both methods were researched and implemented on the computer. The simulation results indicate that there is virtually no difference between the estimates obtained in these two ways (at least for the models used in this paper).

The maximum likelihood estimator avoids not only integration altogether but also the task of computing the maximum eigenvalue and its eigenvector. All it needs is the updated Fourier coefficients of the conditional density, which are obtained through simple algebraic formulas. Therefore, the maximum likelihood estimator is used in comparison with the extended K-B filter, a standard method for spacecraft attitude estimation.

A comparison between the K-B filtering and our maximum likelihood estimator was conducted by E. J. Lefferts of the GSFC/NASA. A. N. Mansfield of the CSTA generated a sequence of 33 star tracker observations. The average body rates were provided every one-third of a second, and the tracker observation was taken every two minutes. The standard deviation of the tracker measurement noises is 20 arcseconds. For such a low noise level, it is known that linearization is a very good approximation. Therefore, it is not surprising that the maximum likelihood estimator is not much better than the K-B filter.

However, the comparison results indicate that the maximum likelihood estimator is almost always better and converges faster than the K-B filter. It is also noted that: (1) The simulated measurement data are in strict accord with the system model, assuming all the true values of the model parameters and the noise statistics are given. (2) There is no random driving term in the state dynamics model for the spacecraft attitude. Under these two conditions in addition to the low measurement noise level mentioned above, even simple-minded estimators can be expected to perform near optimal. But these conditions are usually far from being met in reality.

The simulated examples in Section IX were chosen to test the robustness of our new schemes and represent tougher working conditions than the real ones. The simulation results to be depicted in graphs indicate that both the local integration estimator and the maximum likelihood estimator track the signal nicely.

17

The robustness of an estimator toward uncertainty of system parameters and the random driving term in the state dynamics is perhaps the most important consideration in real world application. While there is every reason to believe that the maximum likelihood estimator is superior in this regard, the issue remains to be resolved in the future.

(14) Convolution of Exponential Fourier Densities and Filtering on the Circle --

While the EFD(n)'s are closed under conditioning, they are not closed under convolution. This deficiency has prevented us from including random driving terms in the signal processes in our results using EFD(n)'s. In this paper, we will present a striking property of the EFD(n)'s; namely, they are almost closed under convolution.

The maximum informational distance, in the sense of Kullback, between the convolution of two EFD(1)'s and its best fit by an EFD(1) is numerically calculated and is 0.00539412984011850. Such a small number indicates that the convolution of any two EFD(1)'s is virtually an EFD(1). Hence it is not surprising that replacing the convolution of two a priori EFD(1)'s with its best fit EFD(1) yields a near optimal estimate, which is almost indistinguishable from the optimal one. All these numerical results are reported in the paper.

The case of EFD(2)'s will also be thoroughly studied in the paper. More details will be submitted to AFOSR as soon as available.

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The results covered by this final report can be put in two categories – continuous-time systems and discrete-time systems. The continuous-time systems considered are bilinear in form. The bilinear systems have been extensively studied in recent years for three primary reasons. First, it has been shown that bilinear systems are feasible mathematical models for large classes of problems of practical importance. Second, bilinear systems provide higher order approximations to nonlinear systems than do linear systems. Third, bilinear systems have rich geometric and algebraic structures which promise a fruitful field of research.

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20. Abstract continued

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